

sidering the fact that $b_0 < 0$, the following stability criteria are obtained in the general form

$$\sum_{i=0}^p \frac{1}{b_i} + 1 < 0, \quad p=0, 1, \dots, n$$

where n is an integer related to the number of admissible functions used in the series representing the elastic displacements. Note that the stability criteria derived in Ref. 3 are more general than those of Ref. 1, and reduce to those of Ref. 1 if the radial booms are omitted.

Dong and Schlack seem to attach considerable importance to infinite series as opposed to finite series. Whether a series is infinite or finite should not be overemphasized, however, as it is seldom necessary to take a large number of terms in the series. Indeed, for the mathematical model in question, Ref. 2 shows that the inclusion of a second mode in the series have virtually no effect on the stability regions, a fact also confirmed by Ref. 1. It is safe to say that in general an infinite series is never summed up for the purpose of producing stability criteria by the assumed modes method, and practical computational considerations demand series truncation at some point.

References

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- ⁴Meirovitch, L. and Calico, R., "A Comparative Study of Stability Methods for Flexible Satellites," *AIAA Journal*, Vol. 11, Jan. 1973, pp. 91-98.
- ⁵Meirovitch, L. and Calico, R., "The Stability of Motion of Satellites with Flexible Appendages," NASA CR-1978, Feb. 1972.

Reply by Author to R. A. Calico and L. Meirovitch

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UNFORTUNATELY, W. N. Dong is not available to participate in this reply to the Technical Comment. Consequently, I will attempt to represent our viewpoints on the issues raised in order to submit this Reply in time for it to appear in the same issue as the Comment.

The authors of the Technical Note¹ under discussion made every attempt to give proper reference to the related publications of Meirovitch and Calico,^{2,3} since these are important papers and we share the high professional regard that exists for their numerous contributions to the elastic satellite dynamics literature. In the preparation of Ref. 1, we were

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aware of numerous papers which involve models of satellites with flexible antennas used for comparing various analytical techniques, including those referred to in the second paragraph of the Technical Comment above. The capability of comparing results with other published research is precisely the reason that a satellite model with flexible antennas was selected for study in Ref. 1.

Although the satellite model in Ref. 1 involves axial antennas only, the motion of the mass center of the satellite due to elastic flexibility is included in the analysis in order to derive infinite series solutions that account for these effects. In the examples discussed in Refs. 2 and 3, the motion of the mass center was assumed negligible. Consequently, the mathematical model as well as the method of analysis in Ref. 1 are sufficiently different from Refs. 2 and 3 that the proposed notion of Ref. 1 being simply a reduced case of these earlier studies is unfounded. The stable regions presented in Fig. 2 of Ref. 1 are somewhat larger than those of Ref. 2 and they are in close agreement with the stability diagrams presented by Kulla,⁴ whose work is in agreement with various studies using different analytical techniques. The erroneous implication in Ref. 1 that the Rayleigh quotient approximation was used throughout Refs. 2 and 3 was unintentional.

The statement made that "Ref. 1 implies that the ability to derive closed-form stability criteria in terms of infinite series represents a new development" is also unfounded. Quite the contrary, since clear reference is made in Ref. 1 to a general procedure for developing closed-form solutions for stability criteria in terms of infinite series by Brown and Schlack,⁶ as early as August 1969. The statement made in Ref. 1 that "most investigators have found it necessary to truncate their assumed modal series before introducing it into the analysis for determining stability criteria" does not exclude the existence of other series solutions such as those presented in Refs. 2 and 3, for example.

Regarding the importance of infinite series solutions, it is well known that theoretical questions and practical considerations related to series truncation have important implications in both research and design. It is now becoming increasingly clear that useful stability criteria can often be derived by using only a very few series terms. However, several authors have discussed fundamental questions regarding modal series truncation error, including an interesting discussion by Meirovitch and Calico in Ref. 3 suggesting a need for increased research attention in this area.

References

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- ²Meirovitch, L. and Calico, R., "The Stability of Motion of Force-Free Spinning Satellites With Flexible Appendages," *Journal of Spacecraft and Rockets*, Vol. 9, April 1972, pp. 237-245.
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- ⁵Brown, D. P. and Schlack, A. L., "Stability of a Spinning Body Containing an Elastic Membrane Via Liapunov's Direction Method," *AIAA Journal*, Vol. 10, Oct. 1972, pp. 1286-1290.
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